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The Symposium took place at Eskişehir-Osmangazi University, Turkey. The Scientific Committee which led the Symposium consisted of A. Akyıldız, K. Arslan, S. Bilir, A. Ergin, M. Ergüt, E. Esin, A. Görgülü, O. Gürsoy, H. Hacısalihoğlu, B. Karakas, B. Karlığa, R. Kaya (Chairman), S. Keles, N. Kuruoğlu, S. Olgun, H. Önsiper, E. Özdamar, A. Özdeğer, A. Salimov, S. Sertöz, C. Tezer, A. Uysal, Z. Yapar. The following are abstracts of talks presented in the symposium.

Nesip Aktan (Eskişehir Osmangazi University, Turkey
A. Ceylan Çöken (Süleyman Demirel University, Turkey)

On weakly symmetric and weakly Ricci-symmetric K-manifolds with Lorentz metric

In this study, we consider weakly symmetric and weakly Ricci-symmetric K-manifolds with Lorentz metric. We find necessary conditions in order that a K-manifold with Lorentz metric be weakly symmetric and weakly Ricci-symmetric.

References: [1] D. Blair, *Contact manifolds in Riemannian Geometry*, Lecture Notes in Math. **509**, Springer, Berlin-Heidelberg-New-York, 1976. [2] K. L. Duggal and Aurel Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer, Dordrecht-Boston-London, 1985. [3] U. C. De, T.Q. Binh and A.A. Shaikh, *On weakly symmetric and weakly Ricci-symmetric K-contact manifolds*, Acta Math. Academiae Paedagogicae Nyíregyháziensis **16** (2000) 65–71.

Sezgin Altay, Füsun Özen (Istanbul Technical University, Turkey)

On weakly and pseudo concircular symmetric structures on a Riemannian manifold

The notions of weakly symmetric manifolds were introduced by [1]. A non-flat Riemannian manifold (M_n, g) $(n > 2)$ is called weakly symmetric if the curvature tensor R_{hijk} satisfies the condition, $\nabla_l R_{hijk} = A_l R_{hijk} + B_h R_{lijk} + B_i R_{hljk} + D_j R_{hilk} + D_k R_{hijl}$, where A, B, D are

1-forms (non-zero simultaneously), [2]. We proved that the defining condition of a weakly concircular symmetric space $(WZS)_n$ can always be expressed as $\nabla_l Z_{hijk} = A_l Z_{hijk} + B_h Z_{lijk} + B_i Z_{hljk} + D_j Z_{hilk} + D_k Z_{hijl}$, [3]. Then, putting $A_l = 2\lambda_l$, $B_l = D_l = \lambda_l$ in this relation, the space is called pseudo concircular symmetric. Some characteristic properties of a hypersurface of $(WZS)_{n+1}$ were obtained. In the last part of this study, an example of the existence of a $(WZS)_n$ was given.

References: [1] L. Tamassy and T.Q. Binh, *On weakly symmetric and weakly projective symmetric Riemannian manifolds*, Coll. Math. J. Bolyai, **50** (1989) 663–670. [2] U.C.De and S.Bandyopadhyay, *On weakly symmetric Riemannian spaces*, Publ. Math. Debrecen **54** (1999) 377–381. [3] F. Özen and S. Altay, *On weakly concircular symmetric spaces*, Math. Pannonica **16** (2005) 29–38.

Aydın Altin (Dokuz Eylül University, İzmir, Turkey)

Ergodicity and stability

In this paper, the definitions, theorems, proofs and figures are intended to be a rather complete and self-contained exposition of the theory of ergodicity and stability, presented in detail and in a form accessible to readers without any prior knowledge of the subjects described. In particular, I improve well-known results of repellor, attractor, ergodicity and stabilities regarding existence via arguments that are simpler than those that can be found in the literature of these areas. I present proofs which are largely self-contained and do not rely on previous theories. In order to give proofs I use the saddle surface S with negative curvature. The elementary geometric and topological properties of the saddle surface S enable the theory to be versatile and clear. A number of tools and issues encountered in the theory of real and dual unit spherical representations are illustrated here in an elementary and simple manner, usually free from difficult theories of the spherical motions. In the manuscript, I also give the topological properties of the unit spherical representation mappings of the saddle surfaces in detail. I believe that my presentations provide a contemporary summary of existing motion theory, ergodicity and stability, as well as some new results of mappings of bundles and bundles of bundles.

References: [1] C. Pugh, *Stable ergodicity*, Bull. Amer. Math. Soc. (N.S.) **41** (2004) 1–41. [2] A. Altın, *Spherical motions and dual Frenet–Serret formulas,* Turkish J. Math., Mat 0406-5 (2005). [3] A. Altın, *Plane mechanism and dual spherical spatial motions,* Math. Balkanica (N.S.) **3–4** (2005).

Arif Amirov (Zonguldak Karaelmas University, Turkey)

Boundary rigity for Riemannian manifold

In this work the problem of reconstructing a Riemannian metric in the bounded domain D of space with the boundary S of class C^3 from the distances between the boundary points

of the domain in a metric g is investigated. A Riemannian metric is called simple, if any two points $x_0, x \in \bar{D}$ can be joined by a unique geodesic of this metric, all points of which, with the exception of possible ends, belong to the domain D. For the points $x_0, x \in S$ let us denote by $H_g(x, x_0)$ the distance between the points x_0, x in the metric g. The function $H_g(x, x_0)$ determined on the set $S \times S$ is called the hodograph of the metric g.

PROBLEM. Determine a metric g in the region D if $H_g(x, x_0)$ is known.

THEOREM. *Let g_1, g_2 be two simple metrics in D and let $H_{g_11}(x, x_0) = H_{g_2}(x, x_0)$. Then the following hold.*

(a) *There exists a diffeomorphism $\varphi : D \to D$, that $\varphi_S = \mathrm{id}$ and $g_2 = \varphi^* g_1$, $\varphi \in C^3(D)$.*
(b) *If $g^k_{1i} = \delta_{1i}$, where $i = 1, 2, ..., n$, $k = 1, 2$, and δ_{1i} is the Kronecker symbol, then the metrics g_1 and g_2 coincide on D.*

References: [1] V.G. Romanov, *Inverse problems of mathematical physics* M. Nauka 1980. [2] R.G. Mukhometov, *Problem of restoring the Riemannian metric*, Sib. Mat. J. **22** (1981) 119–135. [3] A.K. Amirov, *To the problem about the restoration of Riemannian metric*, Dokl. Akad. Nauk **300** (2001) 151–154.

Vedat Asil and Mustafa Yeneroğlu (Firat University, Turkey)

Kinematic modeling for displacements in 3-dimensional Lorentzian space

In this work, coordinate tranformation of sperical displacements is examined in L^3. However, Cayley's formula, Rodriques' equation and Euler's parameters are given in their Lorentzian meaning.

References: [1] J.M. MaCarthy, *An introduction to theoretical kinematics*, The MIT Press Cambridge, Massachusetts London, England, 1990. [2] F.Dillen and W. Kühnel, *Ruled Weingarten surfaces in Mikowski 3-space*, Manuscripta Math. **98** (1999) 307–320. [3] M. Yeneroglu and V. Asil, *On the Geometry of surfaces in spatial motions*, Jour. Inst. Math. Math.&Comp. Sci. **15** 2002.

İsmail Aydemir (Ondokuz Mayis University, Samsun, Turkey)
Keziban Orbay (Ondokuz Mayis University, Amasya, Turkey)

The ruled surfaces generated by Frenet vectors of time-like ruled surface in the Minkowski space $\mathbb{R}^3_1$

In this paper, we study time-like ruled surfaces with space-like directional vector and ruled surfaces generated by Frenet vectors of these surfaces in Minkowski space. We obtain the distribution parameters, the mean curvatures and the Gaussian curvatures of these surfaces. Also, some results and theorems for these surfaces to be developable and minimal are given.

References: [1] O. Kobayashi, *Maximal surfaces in the 3-dimensional Minkowski space,* Tokyo Journal Mathematics **6** (1983) 297–309. [2] I.V. de Woestijne, *Minimal surfaces in the 3- dimensional Minkowski space,* Geometry and Topology of Submanifolds II, Word Scientific, Singapore, (1990) 344–269. [3] S. O'Neill, *Semi-Riemannian Geometry,* Academic Press, New York, 1983.

İsmet Ayhan, A. Ceylan Çöken and Şevket Civelek
(Süleyman Demirel University and Pamukkale University , Turkey)

Horizontal lifts on the tangent bundle

In this study, the components of the Levi-Civita connection $\widetilde{\nabla}$ were computed by using the Sasaki metric $g^S = g^V + g^H$ on the manifold TM. From these components, the horizontal lifts of differential objects on the manifold TM were obtained, and then the VH, CH, HH lifts of differential objects on manifold M were given.

References:[1] İ. Ayhan, *Second order lifting of derivations and tensor fields*, Master Thesis, Pamukkale University, 1997. [2] V. Oproiu, N. Papaghiuc, *On the geometry of tangent bundle of a (pseudo-)Riemannian manifold*, An. Stiint. Univ. Al. I. Cuza Iasi, (S.N.) **44** (1998) 67–83. [3] K. Yano, E.T. Davies, *Metrics and connections in the tangent bundle*, Kodai Math. Semin. Rep. **23** (1971) 493–504.

A. Bayar, S. Ekmekçi (Osmangazi University, Eskişehir, Turkey)

On the taxicab sine and cosine functions

The Taxicab Plane consists of points, lines and angles in Euclidean Plane. In this plane, for points $X = (x_1, y_1)$ and $Y = (x_2, y_2)$, the distance is measured by taxicab metric, $d_T(X, Y) = |x_1 - x_2| + |y_1 - y_2|$. In this study, we develop cosine and sine functions in the taxicab plane by using the reference angle. Also, we give geometrical properties obtained by using these functions.

References: [1] Z. Akça and R. Kaya, *On the norm in higher dimensional taxicab spaces*, Algebras, Groups and Geometry, Hadronic Press (to appear). [2] R. Brisbin and P. Artola, *Taxicab Trigonometry,* Pi Mu Epsilon J. **8** (1984) 89–95. [3] M. Özcan, S. Ekmekçi and A. Bayar, *A note on the variation of taxicab lengths under rotations*, Pi Mu Epsilon J. **11** (2002) 381–384.

Basri Çelik, Süleyman Çiftçi and Atilla Akpinar
(Bursa Uludağ University, Turkey)

Some collineations of a certain class of Moufang Klingenberg planes

In this paper, some collineations of Moufang–Klingenberg planes coordinatized by the altenative ring $\mathcal{A} = \mathbf{A} + \mathbf{A}\varepsilon(\varepsilon^2 = 0)$ which is called the ring of dual numbers constructed on an alternative ring $\mathbf{A}$ are introduced. These collineations are necessary for showing 4-transitivitiy of the collineation group of the MK-plane.

References: [1] C.A. Baker, N.D. Lane and J.W. Lorimer, *A coordinatization for Moufang-Klingenberg planes*. Simon Stevin **65** (1991) 3–22. [2] A. Blunck, *Cross-ratios in Moufang-Klingenberg planes*. Geom. Dedicata **43** (1992) 93–107. [3] S. Çiftçİ, R. Kaya, J.C. Ferrar, *On 4-transitivity in the Moufang plane*. J. Geom. **31** (1988) 65–68.

A. Ceylan Çöken and Ünver Çiftçi
(Isparta Suleyman Demirel University , Turkey)

On degenerate manifolds of constant curvature

Singular semi-Riemannian manifolds of constant curvature are studied by using the Levi–Civita connection whose existence was shown by Kozlov[2]. Since the sectional curvature is independent of the connection chosen, a classificaton of degenerate spaces of constant curvature is given.

References: [1] S.E. Kozlov, *Levi-Civita connections on degenerate pseudo-Riemannian manifolds*, J. Math. Sci. **104** (2001) 1338–1342. [2] D.N. Kupeli,*Degenerate manifolds*, Geom. Dedicata **23** (1987) 259–290. [3] B. O'Neill, *Semi-Riemannian geometry with applications to relativity*. Academic Press, New York, 1983.

Özlem Defterli (Çankaya University, Ankara, Turkey)

Geometrical aspects of superintegrability in two dimensional spaces of non-constant curvature

The problem of when a two-dimensional Riemannian space admits more than one quadratic constant is discussed and the results given by Darboux and Koenigs are listed. By considering the Darboux space of type one, the method of separation of variables in three different coordinate systems is applied to the Schrödinger equation that corresponds to the free Hamilton–Jacobi equation. The last part deals with the potentials that give superintegrability.

References: [1] D. Baleanu, Ö. Defterli, *Killing - Yano tensors and angular momentum*, Czech. Journ. Phys. **54** (2004) 157. [2] E.G. Kalnins, J.M. Kress, P. Winternitz, *Superintegrability in a two dimensional space of non-constant curvature*, J. Math. Phys. **43** (2002) 970. [3] G. Koenigs, *Sur les géodésiques a intégrales quadratiques*, in: *Leçons sur la théorie générale des surfaces*, G. Darboux, (Chelsea Publishing, vol.4, 1972), pp. 368–404.

Ali Deniz (Anadolu University, Turkey)
Andrei V. Ratiu (Istanbul Bilgi University, Turkey)

Periodic trajectories in some cyclic polygons

The problem of the study of the behaviour of a particle moving freely within a plane domain with piecewise smooth boundary, and bouncing at the boundary is called the billiard problem. One of the subjects of study is the existence of periodic trajectories in polygonal

billiards. It is still not yet known whether all polygonal billiards have such trajectories. In the case of triangles, even if it is well known that every acute triangle has a Fagnano trajectory, the general case is still not solved. In this work we show that the necessary and sufficient condition for a quadrilateral Q to have a trajectory hitting all its edges in order, as the Fagnano trajectory does, is that Q should be inscribed in a circle with centre inside Q. A similar result has been obtained for a class of $2n$-gons inscribed in a circle.

References: [1] A. Deniz, *Investigation of trajectories in various classes of billiards*, Ph.D. Thesis, Anadolu University, 2005.

Ali Görgülü, Nesip Aktan (Eskişehir Osmangazi University, Turkey)

On induced framed lightlike hypersurfaces of K-manifolds with Lorentz metric

In this study, K-manifolds were introduced. Also, some results were given for induced lightlike framed hypersurfaces of K-manifolds with Lorentz metric.

References: [1] O'Neill, B. *Semi-Riemannian Geometry with Applications to Relativity*, Academic Press, New York, 1983.

İbrahim Günaltılı (Eskişehir Osmangazi University, Turkey)

Embedding some pseudo-semicomplements in finite projective planes

In a finite linear space $\mathcal{S}$ with v points and $n^2 + n + 1$ lines a pseudo-semicomplement is a set whose parameters are those of the complement of configuration in a finite projective plane such that no line is removed and a sufficient number of lines lost only one point. We show that a pseudo-semicomplement is re-embeddable in the finite projective plane. Using this fact, we also give a new characterization of linear spaces which are the pseudo-semicomplement of some configuration of finite projective planes.

References: [1] İ. Günaltılı, *Pseudo-semicomplements in finite projective planes*, (submitted in Ars combinatoria). [2] İ. Günaltılı, Ş. Olgun, *On the embedding some linear spaces in finite projective planes*, J. Geom. **68** (2000) 96–99. [3] İ. Günaltılı, P. Anapa, Ş. Olgun, *Embedding of complements of some hyperbolic planes*, (to publish in Ars combinatoria).

Armin Herzer (Universität Mainz, Germany)

On representations of chain geometries

For a field K and a K-algebra R we consider the *chain geometry* $\Sigma(K, R)$: the point-set is the projective line over R and the chains are the (projective) K-sublines.

We seek representations of this rather abstract construction in affine or projective spaces. We get a well-known example, the inversive plane, for K the real and R the complex numbers. In the following list (a) is this example and (b) is the general case. We restrict to algebras R of finite dimension over K.

1. Affine chain geometry. (a) The affine inversive plane consists of the affine plane over the real numbers where the chains are all circles and all lines. (b) Considering R as affine space over K, the chains are distinguished affine normal rational curves. The family of these curves is invariant under translations, dilatations and the inversion. This construction is also available more general for Jordan algebras.

2. (a) An inversive plane can be represented on a sphere S in 3-space, where the chains are the intersections of S with planes. (b) For special chain geometries (e.g. R a kinematic algebra), the point-set is a quadric Q and the chains are the intersections of Q with planes.

3. (a) The point-set is an elliptic congruence C (the "points" are the lines belonging to C). The chains are the reguli contained in C. (b) Let V be a K-vector space of dimension $2n$ and let $L(n, V)$ be the set of all subspaces of dimension n. We may consider this as a part of a projective geometry. Now take $L(n, V)$ as point-set and the chains are the n-reguli. Then we have a representation of $\Sigma(K, R)$, where R is the algebra of (n, n)-matrices with entries in K. For any subbalgebra S of R, $\Sigma(K, S)$ is represented by an appropriate subset of $L(n, V)$ closed under n-reguli.

4. Using the Grassmann map applied to 3: In (a) the model of 2(a) ia obtained once more. In the general case (b), a (quasiprojective) variety V is obtained, and the n-reguli are mapped onto (projective) normal rational curves of degree n on V.

5. We shall finish with some remarks on projective chain geometries (rational representations) as generalisations of 2 and 4.

Reference: A.Herzer, *Chain geometries*, in: Handbook of Incidence Geometry, F. Buekenhout, Ed., Elsevier, 1995.

Seyfullah Hizarci and Sheyla Elmas (Ataturk University, Erzurum, Turkey)

On the conform transformation

Examining surface maps and derivative maps in Euclidean space, we have seen that the indicator N of normals obtained from the derivation of the essential normal vector field of N on T, N, B frame-works at any point of a curve on a surface is the same as the vector N. This shows that the map providing the vector N^* from the vector field N of a unit speed curve is a conformal map.

References: [1] H. Hacısalihoğlu, *Differential Geometry*, İnönü University Publications, Malatya, 1983. [2] M. Öztürk, *Elementary Differential Geometry*, Atatrk University Kazım Karabekir Education Faculty Publications, 1985.

Kazım İlarslan (Afyon Kocatepe University, Turkey)

On position vectors of a spacelike W-curve in Minkowski 3-space

A curve α is called a W-curve (or a helix), if it has constant Frenet curvatures. W-curves in the Euclidean space $\mathbb{E}^n$ have been studied intensively. The simplest examples are circles as planar W-curves and helices as non-planar W-curves in $\mathbb{E}^3$.

All W-curves in the Minkowski 3-space are completely classified by Walrave in [1]. For example, the only planar spacelike W-curves are circles and hyperbolas. All spacelike W-curves in the Minkowski space-time $\mathbb{E}_1^4$ are studied in [2]. The examples of null W-curves in the Minkowski space-time are given in [3].

In this paper, we obtain the position vectors of a spacelike W-curve with spacelike, time-like and null principal normal N and by using position vectors we give some characterization for the spacelike W-curve whose image lies on the pseudohyperbolical space $\mathbb{H}_0^2$ in $\mathbb{E}_1^3$.

References: [1] J. Walrave, *Curves and surfaces in Minkowski space*, Ph.D. thesis, K. U. Leuven, Fac. of Science, Leuven, 1995. [2] M. Petrović-Torgašev and E. Šućurović, *W-curves in Minkowski space-time*, Novi Sad J. Math. bf32 (2002) 55–65., [3] W. B. Bonnor, *Null curves in a Minkowski space-time*, Tensor **20** (1969) 229–242.

Mustafa Inç, Mehmet Bektaş, Mahmut Ergüt and Hasan Bulut
(Elazığ Fırat University, Turkey)

Geometrical interpretation and approximate solution of the sine-Gordon equation

In this paper, we obtain the nonlinear sine-Gordon (sG) equation by frenet frame of time-like curve in 3-dimensional Lorentzian space. We used Adomian's decomposition method for solving the sG equation using *Mathematica*. This method in applied mathematics can be an effective procedure to obtain analytic and approximate solutions. The numerical results show that our present method is easier and more accurate than other traditional methods. The method performs well.

References: [1] M.Inc and Y.Cherruault, *A new approach to solve a diffusion-convection problem*, Kybernetes **31** (2002) 536. [2] M.Inc, *Adomian decomposition method for optimal control problems of Biomedical systems*, Hacettepe Bull. Natural Sci. Eng. **30** (2001) 69.

Gülay Karadoğan (Middle East Technical University, Turkey)

The moduli of surfaces admitting smooth genus two fibrations over elliptic curves

We study the structure, deformations and the moduli spaces of complex projective surfaces admitting genus two fibrations over elliptic curves. We observe that a surface admitting a smooth fibration is elliptic and these fibrations are smooth apart from two double fibres. Using a natural polarization on these surfaces, we employ results on the moduli of polarized elliptic surfaces to construct moduli spaces of these smooth fibrations.

References: [1] W. K. Seiler, *Global moduli for elliptic surfaces with a section*, Compositio Math. **62**, (1972) 169–185. [2] W. K. Seiler, *Global moduli for polarized elliptic surfaces*, Compositio Math. **62** (1972) 187–213. [3] R. Friedman, J.W. Morgan, *Smooth Four-Manifolds and Complex Surfaces*, Springer-Verlag, 1991.

Baki Karliğa (Gazi University, Turkey)

Some geometric interpretations for k-order vertex angles of hyperbolic simplices

The orthogonal projection and the distance of a point to a k-plane in H^n and S^n are calculated. Using the distance of a point to a k-plane, we obtain some geometrical interpretations for sines of k-order vertex angles of an n-simplex in these spaces.

References: [1] B. Karlığa, *Edge matrix of hyperbolic simplices*, Geom. Dedicata, **109** (2004) 1–6. [2] B. Karlığa, A.T.Yakut, *Vertex angles of simplices in hyperbolic space H^n*, Submitted for Publication, 2005. [3] E.B. Vinberg,*Geometry II*, Encyclopaedia of Mathematical Sciences Vol 29, Springer, Berlin,1993.

R. Kaya, Ö. Gelişgen, S. Ekmekçi and A. Bayar
(Eskişehir Osmangazi University, Turkey)

Group of isometries of the CC-Plane

In this work, it is shown that the group of isometries of the plane with respect to the Chinese Checkers metric is the semi-direct product of the dihedral group D_8 and $T(2)$, where D_8 is the symmetry group of the regular octagon and $T(2)$ is the group of all translations of the plane. Also, some proporties of the CC-plane are studied and an area formula for a triangle is given.

References: [1] D.J. Schattschneider, *The taxicab group*, Amer. Math. Monthly 91 (1984), 423–428. [2] R. Kaya, Ö. Gelişgen, S. Ekmekçi, A. Bayar, *Group of isometries of CC-plane*, Eskişehir Osmangazi University, Department of Mathematics Preprint Nr. 1 2004.

İsmail Kocayusufoğlu (Eskişehir Osmangazi University,Turkey)
Tuba Ada (Eskişehir Anadolu University, Turkey)

On the iso-taxicab trigonometry

Iso-taxicab geometry was defined by K.O. Sowell. It is known that iso-taxicab geometry is another non-Euclidean geometry similar to taxicab geometry; the main difference in iso-taxicab geometry is the metric which is defined as follows:

$$d_I((x_1, y_1), (x_2, y_2)) = \begin{cases} \text{(i)} & |x_1 - x_2| + |y_1 - y_2|, & I - IV & \text{orientation} \\ \text{(ii)} & |y_1 - y_2|, & II - V & \text{orientation} \\ \text{(iii)} & |x_1 - x_2|, & III - VI & \text{orientation} \end{cases}$$

In this study, we discuss the iso-taxicab trigonometry. First, sine, cosine, tangent and cotangent functions are defined. Finally we construct addition-subtraction and half-angle formulas, and the table for iso-taxicab trigonometric functions.

References: [1] E.F. Krause, *Taxicab Geometry*, Addison–Wesley, Menlo Park, NJ, 1975. [2] K.O. Sowell, *Taxicab geometry - a new slant*, Mathematics Magazine **62** (1989) 238–248. [3] K.Thompson, K. Dray, *Taxicab angles and trigonometry*, Pi Mu Epsilon **11** (2000) 87–96.

İsmail Kocayusufoglu and Şükrü Ünver (Eskişehir Osmangazi University, Turkey)

Triangles and the isoperimetric theorem

The problem of the isoperimetric inequality is not new. The isoperimetric theorem can be stated as follows:"Among all closed curves with the same perimeter in the plane, the circle has the greatest area." In this study, this theorem is discussed for triangles and smooth polygons.

Reference: [1] A. Miller, *The isoperimetric problem: a motivated look at the calculus of variations*, 2000.

Gülay Koru Yücekaya (Gazi University, Turkey)

Holditch's theorem in Lorentz 3-space

Some properties and theorems for the closed spherical movement in Euclidean 3−space were given by Hacısalihoğlu (1971). In this study, the Lorentz sphere in 3-dimensional space is considered, and some applications of Holditch's theorem on Lorentz circles on the Lorentz sphere are given. The correspondence with results in Euclidian 3−space are explained.

References:[1] H.H. Hacısalihoğlu, *On closed spherical motions*. Q. Appl. Math. **29** (1971) 269–276. [2] E. Özyilmaz and Y. Yayli, *O. Bonnet integral formula and some theorems in Minkowski space*, Hydronic Journal **15** (2000) 397–414. [3] I.M. Yaglom, *A Simple Non-Euclidean Geometry and Its Physical Basis*, New York, 1979

Hans-Joachim Kroll (Tecchnische Universität München, Germany)

On the Gordon bound for permutation decoding sets

One of the pivotal problems of coding theory is decoding. For applications, quick algorithms are needed. For this purpose, F.J. Mac Williams developed in 1964 the technique of permutation decoding. This technique uses a subset of the automorphism group of the code, a so-called permutation decoding set. A *permutation decoding set*, *PD-set* for short, for a t-error-correcting code C is a set S of automorphisms of the code so that every possible error vector of weight t or less can be moved by some member of S out of the information positions.

Since the permutation decoding algorithm is more efficient the smaller the size $|S|$ of the PD-set S it is important for the applications to find small PD-sets. A lower bound on the size of a PD-set is given by D.M. Gordon.

There are examples for PD-sets, but up to now there is no known method to find PD-sets; and in many cases the size of the PD-set is much larger than that of the Gordon bound. Here the question arises whether the Gordon bound is in any case sharp. This means the following: Is there in any case a PD-set S (in the corresponding symmetric group) such that the size $|S|$ is the Gordon bound?

To handle this problem we introduce the notion of an antiblocking system. The properties of such an antiblocking system may help to find PD-sets and to prove whether a PD-set of given size may exist or not. Indeed, based on our results concerning antiblocking systems we will present examples where the Gordon bound is not sharp.

This is a joint work with Rita Vincenti.

References: [1] H.-J. Kroll, R. Vincenti, *PD-sets for the codes related to some classical varieties*, Discrete Math. **301** (2005) 89–105. [2] H.-J. Kroll, R. Vincenti, *Antiblocking systems and PD-sets*, to appear in Discrete Math.

Aytaç Kurtuluş and Şükrü Olgun (Eskişehir Osmangazi University, Turkey)

$(k; n)$-arcs in the finite inversive plane

In this work, a definition of a $(k; n)$-arc in a finite inversive planes is given analogous to a (k, n)-arc in a finite projective plane. Some combinatorial properties are obtained and maximal $(k; n)$-arcs are briefly discussed.

References: [1] A. Sonnino, *k-archi nei piani di Benz,* Note di Matematica **15** (1995) 111–119. [2] F. Buekenhout, *Handbook of Incidence Geometry,* Amsterdam, 1995.

Bedia Akyar Møller (Dokuz Eylül University, Turkey)

Prism complexes in Chern-Simons theory

This work attempts to find the variation of the Chern-Simons class for a given bundle over a simplicial set S with a connection w. For a given simplicial set S, the prismatic subdivision $P_pS_{q_0,\dots,q_p} = S_{q_0+\cdots+q_p+p}$, which is a prismatic complex, is used to find the required variation. An appropriate classifying space and required transition functions can be defined. So the classifying map can be defined by the transition functions. Finally by working at the chain level, a cycle can be obtained and Chern-Simons class on it evaluated.

References: [1] B. Akyar, *Lattice gauge field theory*, Phd. Thesis, Århus University, 2002. [2] B. Akyar, *The Chern-Simons character of a lattice gauge field*, Quantum Topology (1993) 244–291. [3] S.S. Chern, J. Simons, *Characteristic forms and geometric invariants*, Ann. of Math. **99** (1974) 48–69.

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On the ruled surfaces in the Minkowski 3-space $\mathbb{R}^3_1$

S. Izumiya and N. Takeuchi [2] obtained some characterizations for ruled surfaces. H.H. Hacısalihoğlu and A. Turgut [3] defined timelike ruled surfaces and obtained some characterizations.

In this study, using the results of [1,2], we investigate cylindrical helices and Bertrand curves as curves on timelike ruled surfaces in Minkowski 3-space $\mathbb{R}^3_1$.

References: [1] H. Balgetir, M. Bektaş and M. Ergüt, *Bertrand curves for nonnull curves in 3-dimensional Lorentzian space*, Hadronic Journal **27** (2004) 229–236. [2] S. Izumiya and N. Takeuchi, *Special curves and ruled surfaces*, Beiträge Algebra Geom. **44** (2003) 203–212. [3] A. Turgut and H.H. Hacısalihoğlu, *Timelike ruled surfaces in the Minkowski 3-space-II*, Tr. J. of Mathematics **22** (1998) 33–46.

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Weakly symmetric Riemannian manifolds with semi-symmetric metric connection

Let $M_n(\nabla, g)$ be a differentiable manifold of class C^∞. If the condition $\nabla^* g = 0$ is satisfied, such a linear connection ∇^* is said to be a metric connection. If this linear connection satisfies the relation $\nabla^*_X Y = \nabla_X Y + w(Y)X - g(X, Y)U$ then this connection ∇^* is called semi-symmetric. We denote the curvature tensor of this connection by $R^*(X, Y)Z = R(X, Y)Z - P(Y, Z)X + P(X, Z)Y - g(Y, Z)AX + g(X, Z)AY$, where $R(X, Y)Z$ is the

curvature tensor of ∇ and P is a tensor field of type $(0, 2)$ defined on M by $P(X, Y) = (\nabla_X w)Y - w(X)w(Y) + \frac{1}{2}w(U)g(X, Y)$ and $g(AX, Y) = P(X, Y)$, where A is a tensor field of type (1,1). In this study, we have shown that the definition of a weakly semi-symmetric metric connection can always be expressed in the form $\nabla_l R^*_{hijk} = a_l R^*_{hijk} + b_h R^*_{lijk} + b_i R^*_{hljk} + d_j R^*_{hilk} + d_k R^*_{hijl}$, where a, b, c, d are 1-forms that are non-zero simultaneously. As in [1], we take $ds^2 = \phi(dx^1)^2 + k_{\alpha\beta}dx^\alpha dx^\beta + 2dx^1 dx^n$, where $[k_{\alpha\beta}]$ is a symmetric non-singular matrix.

In this paper, we have chosen special 1-forms a_i, b_i, d_i and have determined the metric when the space is $(WSS)_n$. It has then been shown that this space satisfies some special cases.

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On a class of generalized recurrent contact metric manifolds

In this study, we consider generalized recurrent, generalized Ricci-recurrent and generalized concircular recurrent contact metric manifolds, ξ, with k-nullity distribution. We show that there exist no generalized recurrent and generalized Ricci-recurrent contact metric manifolds, unless $k\alpha + \beta$ is everywhere zero. Also, we find the characterizations of scalar curvatures of generalized recurrent and concircular recurrent manifolds of such type.

References: [1] D.E. Blair, T. Koufogiorgos, B.J. Papantoniou, *Contact metric manifolds satisfying a nullity condition*, Israel J. Math. **91** (1995) 189–214. [2] U.C. De, N. Guha, *On generalized recurrent manifolds*, Proc. Math. Soc. **7** (1991) 7–11. [3] Q. Khan, *On generalized recurrent Sasakian manifolds*, Kyungpook Math. J. **44** (2004) 167–172.

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On some invariant characteristic properties of Möbius transformations

Möbius transformations (linear fractional transformations) are the (directly) conformal homeomorphisms of the extended complex plane $\mathbb{C}\cup\{\infty\}$. A Möbius transformation T has the form,

$$T(z) = \frac{az + b}{cz + d}, \quad a, b, c, d \in \mathbb{C} \text{ and } ad - bc \neq 0.$$

These transformations form a group under composition. In this study, the connections between some invariant characteristic properties of Möbius transformations and the

notions of cross-ratio and n-transitivity of the group of Möbius transformations are considered.

References: [1] A.F. Beardon, D. Minda, *Sphere-preserving maps in inversive geometry*, Proc. Amer. Math. Soc. **130** (2002) 987–998. [2] H. Haruki, T.M. Rassias, *A new characteristic of Mö bius transformations by use of Apollonius quadrilaterals*, Proc. Amer. Math. Soc. **126** (1998) 2857–2861. [3] P. Niamsup, *A note on the characteristics of Möbius transformations*, J. Math. Anal. Appl. **248** (2000) 203–215.

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On the torus cobordant cohomology spheres

Let G be a compact Lie group. In 1960 P. A. Smith asked the following question; "Is it true that for any smooth action of G on a homotopy sphere with exactly two fixed points, the tangent G-modules at these two points are isomorphic?" From a result due to Atiyah and Bott, Smith's question has a positive answer for $\mathbb{Z}_p$ and therefore for connected Lie groups. In this work, we prove that two linear actions on S^n which are smoothly c-cobordant (cobordism in which inclusion of each boundary component induces isomorphisms in $\mathbb{Z}$-cohomology) must be equivalent. As a corollary we obtain a proof of the Smith equivalence problem for connected Lie groups using a different method.

References: [1] M.F. Atiyah, R.A. Bott, *Lefschetz fixed point formula for elliptic complexes: II. Applications*, The Annals of Mathematics **88** (1968), 451–491. [2] D. Golber, *The cohomological description of a torus action*, Pacific J. Math. **46** (1973) 149–154. [3] E. Straume, *The integral weight system for torus actions on spheres with no fixed point*, Math. Scand. **66** (1990) 91–109.

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General equation for CC-conics and focus-diractrix CC-conics

Two-foci CC-conics have been studied in [1], [2]. In this work a general equation for a CC-conic is given and the focus-directrix CC-conics are studied. Two-foci and focus-directrix definitions give different figures.

References: [1] M. Turan, M. Özcan, Two foci Chinese Checker ellipses, International Journal of Pure and Applied Mathematics, Volume 16, No.1, 119–127 (2004). [2] M. Turan, M. Özcan, *Two foci Chinese Checker hyperbolas*, International Journal of Pure and Applied Mathematics (to appear)

Idzhad Sabitov (Moscow State University, Russia)

On some recent progress in the metric theory of polyhedra

A famous theorem of Cauchy (1813) affirms that any two convex polyhedra in three-space with the same combinatorial structure and metric are congruent. As a consequence of this

theorem we have non-flexibility of any convex polyhedron P, that is, the non-existence of a continuous deformation of P preserving its faces as rigid plane polygons. After more than 160 years the existence of a flexible polyhedron was shown [1]. It turned out that the volume of any known flexible polyhedron remains constant during the flexion and this was a reason to suppose that the property of invariance of volume is valid for all flexible polyhedra. This conjecture was proved in 1996 [2] but the essential result consisted in *the method* of proof, which related metric characteristics of a polyhedron P to some polynomial equations depending only on the combinatorial structure and lengths of edges of P. This approach indicated an algorithmical solution to the problem of isometrical realization in 3-space of a given polyhedral metric [3] and, finally, to prove, for convex polyhedra, that any function f in coordinates of vertices of a simplicial polyhedron P, invariant relatively to motions, is a root of some polynomial equation whose degree does not exceed 2^N, where N is the number of edges of P.

Some results for cyclic polygons are also suggested.

References: [1] R. Connelly, Publ. Math. IHES **47** (1977) 333–338. [2] I. Sabitov, Discrete Comput. Geom. **20** (1998) 405–425. [3] I. Sabitov, Isvestia Ros. Ac. Nauk. **43** (2002) 159–172. [4] I. Par and M. Fedorchuk, to appear in Duke Math. J.

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Semi-Riemannian submersions

In this presentation, we give fundamental equations of a Riemannian submersion with some explicit examples and generalize Theorem 3 and Lemma 4 in [3] with respect to the Riemannian curvature $\mathfrak{R}$ of the total space.

References: [1] B. O'Neill, *The fundamental equations of a submersion*, Mich. Math. J. **13** (1966) 459–469. [2] A. Gray, *Pseudo-Riemannian almost product manifolds and submersions*, J. Math. Mech. **16** (1967) 715–737. [3] M.A. Magid, *Submersions from anti-de sitter space with totally geodesic fibers*, J. Differential Geom. **16** (1981) 323–331.

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On minimal index of generalized time-like ruled surfaces

In this paper, we introduce the minimal index of $(k+1)$-dimensional time-like ruled surfaces in $\mathbb{R}^n_1$. Relations between minimal indices of these ruled surfaces are also given.

References: [1] I. Aydemir, *Time-like ruled surfaces in the Minkowski spaces* $\mathbb{R}^n_1$, Int. J. Apl. Math. **10** (2002) 149–159. [2] B. O'Neill, *Semi-Riemannian geometry*, Academic Press, New York, London, 1983.

[3] M. Tosun, *On Time-like hyperruled surfaces in the Minkowski n-space*, in: Recent Advances in the Applied and Theoretical Mathematics, Math. Comput. Sci. Eng., World Sci. Eng. Soc. Press, Athens, 2000, pp. 122–127.

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Dual Lorentzian spherical motions

In this paper we first introduce one-parameter dual Lorentzian spherical motions. In addition we give the relations between the absolute, relative and sliding velocities of these motions. The last part of this study is devoted to the equations describing the sliding velocity.

References: [1] G.S. Birman, K. Nomizu, Trigonometry in Lorentzian Geometry, Am. Math. Mont. **91** (1984) 543–549. [2] H. R. Müller, Kinematik Dersleri, Ankara University, Fen Fakültesi Yayınları, No 27, 1963. [3] Y. Yayli, A. Caliskan and H. H. Ugurlu, The E. Study Maps of Circles on Dual Hyperbolic and Lorentzian Unit Spheres H_0^2 and S_1^2, Math. Proc. Royal Irish Acad. **102A** (2002) 37–47.

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Models of hyperbolic space and vertex angles of hyperbolic simplices in the Hyperbolic Model

In this study, we first present five models of hyperbolic space. Then, by using the properties of Gramian, generalized vector cross product and determinant, we compute the vertex angles of spherical and polar spherical simplices and give relations among them.

References: [1] Feng Luo, *On a problem of Fenchel*, Geom. Dedicata **64** (1997) 277–282. [2] B. Karlığa, *Edge matrix of hyperbolic simplices* , Geom. Dedicata **109** (2004) 1–6.

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The theorems of Stewart, Ceva and Menelaus for geodesic triangles on the unit hyperbolic dual sphere $\tilde{\mathbb{H}}_0^2$ of the dual Lorentzian 3-space $\mathbb{D}_1^3$

In this paper, we state and prove the dual hyperbolic theorems of Stewart, Ceva and Menelaus for geodesic triangles on the dual hyperbolic unit sphere $\tilde{\mathbb{H}}_0^2$ in the dual Lorentzian space $\mathbb{D}_1^3$. We also give the Lorentzian geometric interpretations of these theorems in the Minkowski 3-space $\mathbb{R}_1^3$.

References: [1] E. Study, *Geometrie der Dynamen*, Leipzig, 1903. [2] H.H. Uğurlu, H. G ündoğan, *The cosine hyperbolic and sine hyperbolic rules for dual hyperbolic spherical trigonometry*, Mathematical and Computation

Applications **5** (2000) 185–190. [3] H.H. Uğurlu, M. Kazaz and A.özdemir, *Fundamental formulas for geodesic triangles on the dual hyperbolic unit sphere* $\mathbb{H}_0^2$ *and Lorentzian geometrical interpretations*, Hadronic Journal Supplement **18** (2003) 369–386.

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On D-recurrent spaces with semi-symmetric recurrent-metric connection

Let (M_n, g) be an n-dimensional Riemannian space $(n > 2)$ with the Riemannian connection ∇ and the semi-symmetric recurrent-metric connection D. The semi-symmetric recurrent-metric connection on M_n was introduced in [1] and some properties of the connection were studied in [1-3].

In this work, we introduce D-recurrent spaces with semi-symmetric recurrent-metric connection denoted by (M_n, g, D) and we obtain some properties of the curvature tensor L^m_{ijk} of (M_n, g, D). Also, an example of such a space is given.

References: [1] K. Yano, *On semi-symmetric metric connection*, Rev. Roumanie Math. Pures Appl. **15** (1970) 1579–1586. [2] Y.X. Liang, *On semi-symmetric and metric-recurrent connection*, Journal of Xiamen University (natural science) **27** (1988) 248–253. [3] S.A. Uysal and R. Ö. Doğan, *On recurrent spaces with semi-symmetric Singh connection*, Indian J. Pure Appl. Math **34** (2003) 1061–1065.

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On the curvatures of rotation surface in 3-dimensional dual space

In this paper, rotation surfaces are defined in 3-dimensional dual space. Curvatures of these surfaces are calculated with respect to the dual metric. Then, the relation between real and dual parts of these curvatures is examined.

References: [1] G.R. Veldkamp, *On the use of dual numbers, vectors and matrices in instantaneous, spatial kinematics*, Mech. Mach. Theory **11** (1976) 141–156. [2] V. Brodsky and M. Shoham, *Dual numbers representation of rigid body dinamics*, Mech. Mach. Theory **34** (1999) 975–991. [3] V. Asil, *Dual exponential transformations and high-order derivatives*, Inter. J. of Pure and Applied Math. **5** (2004) No.4.

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Lightlike hypersurface of a semi-Riemannian manifold admitting semi-symmetric metric connection.

In this paper, we study lightlike hypersurfaces of a semi-Riemannian manifold admitting a semi-symmetric metric connection. We give the equations of Gauss and Codazzi. Then we obtain some conditions on the Ricci tensor to be symmetric.

References: [1] A. Bejancu, *Null hypersurfaces in semi-Euclidean space*, Saitama Math. J. **14** (1996) 25–40. [2] K.L. Duggal and R. Sharma, *Semi-symmetric metric connection in a semi-Riemannian manifold*, Indian J. Pure appl Math. **17** (1986) 1276–1282. [3] T. Imai, *Hypersurfaces of a Riemannian manifold with semi-symmetric metric connection*, Tensor, N.S. **23** (1972) 300–306.

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An integral geometry problem on three dimensional space

In this work, the existence, uniqueness and stability for the inverse problem for the equation $L_1 u \equiv v\nabla_x u = \lambda$ in a region Ω_2 is investigated. Here,

$$
\begin{aligned}
&L_1 u \equiv v\nabla_x u = u_{x_1}\cos\varphi_1 + u_{x_2}\sin\varphi_1\cos\varphi_2 + u_{x_3}\sin\varphi_1\sin\varphi_2 = \lambda,\\
&\Omega_2 = \{(x, v) : x \in D,\ v \in S^2\},\\
&\qquad D \text{ is a bounded region of } \mathbb{R}^3,\ S^2 \text{ is a unit sphere},\\
&v = \{(\cos\varphi_1, \sin\varphi_1\cos\varphi_2, \sin\varphi_1\sin\varphi_2)\},\\
&\varphi = (\varphi_1, \varphi_2),\ \varphi_1 \in (0, \pi),\ \varphi_2 \in (0, 2\pi).
\end{aligned}
$$

This problem is investigated using Galerkin's method.

References: [1] A. K. Amirov, *Existence And uniqueness theorems for the solution of an inverse problem for the transport equation*. Siberian Mathematical Society (1986). [2] M. M. Lavrent'ev, V.G. Romanov, and S. P. Shishatskii, *Ill-posed problems of mathematical physics and analysis*. Amer. Math. Soc. (1986). [3] M. M. Lavrent'ev and Y. E. Anikonov, *On a class of problems of integral geometry*, Dokl. Akad. Nauk SSSR **176** (1967) 1240–1241.

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On an indefinite Kaehler manifold with complex semi-symmetric metric connection

In this paper, we define a complex semi-symmetric metric connection on an indefinite Kaehler manifold and study properties of the Riemannian curvature tensor of the manifold with respect to this connection. Then, we obtain the relations between the Ricci curvatures and the scalar curvatures with respect to this connection and the Levi–Civita connection. Finally, we give a condition under which the Ricci tensor with respect to a complex semi-symmetric metric connection is symmetric.

References: [1] M. Barros, A. Romero, *Indefinite Kaehler Manifolds,* Math. Ann. **261** (1982) 55–62. [2] T. Sakaguchi, *On complex semi-symmetric F-connection,* Hokkaido Math. J. **5** (1976) 218–226. [3] K. Yano, *On semi-symmetric metric connection*, Rev. Roumaine Math. Pures Appl. **15** (1970) 1579–1586.